№282

$$a)\int\frac{xdx}{(x^2+4)^6}=\left|\begin{array}{l} t=x^2+4\\ dt=(x^2+4)'dx=2xdx\\ xdx=\frac{dt}{2}\end{array}\right|=\int\frac{\frac{dt}{2}}{t^6}=\frac{1}{2}\int\frac{dt}{t^6}=\frac{1}{2}\frac{t^{-5}}{-5}+C=-\frac{1}{10t^5}+C=-\frac{1}{10(x^2+4)^5}+C$$

Проверка:

$$\left(-\frac{1}{10(x^2+4)^5}+C\right)'=-\frac{1}{10}(-5)(x^2+4)^{-6}(x^2+4)'+0=\frac{1}{2(x^2+4)^6}*2x=\frac{x}{(x^2+4)^6}.$$

$$б)\int e^x\ln(1+3e^x)dx=\left[\begin{array}{ll} u=\ln(1+3e^x) & \\ dV=e^xdx & ;\quad du=\dfrac{3e^xdx}{1+3e^x}\\ V=e^x & \end{array}\right]=e^x\ln(1+3e^x)-\int e^x\frac{3e^xdx}{1+3e^x}=e^x\ln(1+3e^x)-$$

$$-\int\frac{1+3e^x-1de^x}{1+3e^x}=e^x\ln(1+3e^x)-\int\left(1-\frac{1}{1+3e^x}\right)de^x=e^x\ln(1+3e^x)-e^x+\frac{1}{3}\int\frac{d(1+3e^x)}{1+3e^x}=$$

$$=e^x\ln(1+3e^x)-e^x+\frac{1}{3}\ln(1+3e^x)+C.$$

Проверка

$$\left(e^x\ln(1+3e^x)-e^x+\frac{1}{3}\ln(1+3e^x)+C\right)'=e^x\ln(1+3e^x)+\frac{e^x}{1+3e^x}*3e^x-e^x+\frac{1}{3}\frac{3e^x}{1+3e^x}=$$

$$=e^x\ln(1+3e^x)+\frac{3e^{2x}-e^x-3e^{2x}+e^x}{1+3e^x}=e^x\ln(1+3e^x).$$

$$в)\int\frac{2x^2-3x+1}{x^3+1}dx=I$$

$$\frac{2x^2-3x+1}{x^3+1}=\frac{2x^2-3x+1}{(x+1)(x^2-x+1)}=\frac{A}{x+1}+\frac{Bx+C}{x^2-x+1}$$

$$\begin{array}{l|l} x^2 & A+B=2\\ x^1 & -A+B+C=-3\\ x^0 & A+C=1\end{array}$$

$A=2-B$

$-2+B+B+C=-3$

$2B+C=-1$

$C=-1-2B;\qquad A-1-2B=1$

$2-B-1-2B=1$

$3B=0\Rightarrow B=0;C=-1;A=2.$

$$I = 2\int \frac{dx}{x+1} - \int \frac{dx}{\left(x-\frac{1}{2}\right)^2+\frac{3}{4}} = 2\ln|x+1| - \frac{2}{\sqrt{3}} arctg\frac{2x-1}{\sqrt{3}} + C.$$

$$г)\int \frac{dx}{\sin x + tgx} = \left[\begin{matrix} \cos x = t \\ \sin x = \sqrt{1-t^2} \\ tgx = \dfrac{\sqrt{1-t^2}}{t} \\ dx = -\dfrac{dt}{\sqrt{1-t^2}} \end{matrix}\right] = -\int \frac{dt}{\sqrt{1-t^2}} \frac{1}{\left(\sqrt{1-t^2}+\dfrac{\sqrt{1-t^2}}{t}\right)} = -\int \frac{tdt}{(1-t^2)(t+1)} = I$$

$$\frac{t}{(1-t^2)(t+1)} = \frac{A}{1-t} + \frac{B}{1+t} + \frac{C}{(1+t)^2} = \frac{A(1+2t+t^2)+B(1-t^2)+C(1-t)}{(1-t^2)(t+1)}.$$

$$I = 2\int \frac{dx}{x+1} - \int \frac{dx}{\left(x-\frac{1}{2}\right)^2+\frac{3}{4}} = 2\ln|x+1| - \frac{2}{\sqrt{3}} arctg\frac{2x-1}{\sqrt{3}} + C.$$

$$г)\int \frac{dx}{\sin x + tgx} = \left[\begin{matrix} \cos x = t \\ \sin x = \sqrt{1-t^2} \\ tgx = \dfrac{\sqrt{1-t^2}}{t} \\ dx = -\dfrac{dt}{\sqrt{1-t^2}} \end{matrix}\right] = -\int \frac{dt}{\sqrt{1-t^2}} \frac{1}{\left(\sqrt{1-t^2}+\dfrac{\sqrt{1-t^2}}{t}\right)} = -\int \frac{tdt}{(1-t^2)(t+1)} = I$$

$$A = B$$
$$2B - C = 1$$
$$C = 2B - 1$$

$$\begin{array}{l|l} t^2 & A - B = 0 \\ t^1 & 2A - C = 1 \\ t^0 & A + B + C = 0 \end{array} \Rightarrow \quad B + B + 2B = 1 \Rightarrow B = \frac{1}{4}; A = \frac{1}{4}$$

$$C = \frac{2}{4} - 1 = \frac{1}{2} - 1 = -\frac{1}{2}.$$

$$г) \int \frac{dx}{\sin x + tgx} = \int \frac{dx}{\sin x + \frac{\sin x}{\cos x}} =$$

$$t = tg\frac{x}{2}$$

$$x = 2arctgt$$

$$dx = \frac{2dt}{1+t^2} \qquad \cos x = \frac{1-t^2}{1+t^2}$$

$$= \int \frac{\cos x dx}{\sin x(\cos x + 1)} = \int \frac{\frac{1-t^2}{1+t^2} * \frac{2dt}{1+t^2}}{\frac{2t}{1+t^2} * \left(\frac{1-t^2}{1+t^2} + 1\right)} = \int \frac{\frac{1-t^2}{(1+t^2)} dt}{\frac{t}{1+t^2}\left(\frac{1-t^2+1+t^2}{1+t^2}\right)} = \int \frac{1-t^2}{(1+t^2)^2} * \frac{(1+t^2)}{t} * \frac{1+t^2}{2} dt =$$

$$= \frac{1}{2}\int \frac{1-t^2}{t} dt = \frac{1}{2}\left(\frac{1}{t} - t\right) + C = \frac{1}{2}\left(\ln t - \frac{t^2}{2}\right) + C = \frac{1}{2}\ln\left(tg\frac{x}{2}\right) - \frac{1}{4}tg^2\frac{x}{2} + C.$$

№292

$$\int_2^{12} \sqrt{x^3 + 9} dx = I$$

$x_0 = 2, x_1 = 3, x_2 = 4, x_3 = 5, x_4 = 6, x_5 = 7, x_6 = 8, x_7 = 9, x_8 = 10, x_9 = 11, x_{10} = 12.$

$y_0 = \sqrt{8+9} = 4{,}123, y_1 = 6, y_2 = 8{,}544, y_3 = 11{,}576, y_4 = 15, y_5 = 18{,}762, y_6 = 22{,}825, y_7 = 27{,}166,$
$y_8 = 31{,}765, y_9 = 36{,}606, y_{10} = 41{,}677.$

По формуле Симпсона:

$$I = \frac{h}{3}[y_0 + y_{10} + 4(y_1 + y_3 + y_5 + y_7 + y_9) + 2(y_2 + y_4 + y_6 + y_8)] =$$

$$= \frac{1}{3}[4{,}123 + 41{,}677 + 4(5 + 11{,}576 + 18{,}762 + 27{,}166 + 36{,}606) + 2(8{,}544 + 15 + 22{,}825 + 31{,}765)] =$$

$$= \frac{1}{3}[45{,}8 + 400{,}44 + 156{,}268] = 200{,}836.$$

№302

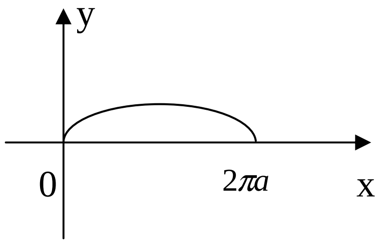
$$\int_{-\infty}^{-3}\frac{xdx}{(x^2+1)^2}=\lim_{a\to-\infty}\int_a^{-3}\frac{xdx}{(x^2+1)^2}=\lim_{a\to-\infty}\frac{1}{2}\int_a^{-3}\frac{d(x^2+1)}{(x^2+1)^2}=\frac{1}{2}\lim_{a\to-\infty}\int_a^{-3}\frac{d(x^2+1)}{(x^2+1)^2}=\frac{1}{2}\lim_{a\to-\infty}\left(-\frac{1}{x^2+1}\right)\Bigg|_a^{-3}=$$

$$=-\frac{1}{2}\lim_{a\to-\infty}\left(\frac{1}{10}-\frac{1}{a^2+1}\right)=-\frac{1}{2}\left(\frac{1}{10}-0\right)=-\frac{1}{2}*\frac{1}{10}=-\frac{1}{20}.$$

Несобственный интеграл сходится

№312

$$\begin{cases} x=a(t-\sin t) \\ y=a(1-\cos t)\end{cases}\quad 0\le t\le 2\pi$$



Для нахождения площади воспользуемся формулой:

$$S=\int_{t_1}^{t_2} y(t)x'(t)dt$$

$t_1=0, t_2=2\pi$

$y(t)=a(1-\cos t)$

$x'(t)=a(1-\cos t)$

$$S=\int_0^{2\pi} a^2(1-\cos t)^2dt=a^2\int_0^{2\pi}(1-2\cos t+\cos^2 t)dt=a^2(t-2\sin t)\Big|_0^{2\pi}+\frac{a^2}{2}\int_0^{2\pi}(1+\cos 2t)dt=$$

$$=a^2*2\pi+\frac{a^2}{2}\left(t+\frac{1}{2}\sin 2t\right)\Bigg|_0^{2\pi}=2\pi a^2+\frac{a^2 2\pi}{2}=2\pi a^2+\pi a^2=3\pi a^2(\text{кв.ед.}).$$